Grade 8 Math

Unit 5 – Percents, Ratios and Rates Study Guide

Percent means out of 100.

Ex. 37% of our students love apples means 37 out of every 100 students love apples.

A percent can also be represented as a fraction or a decimal or a ratio.

Ex.

<table>
<thead>
<tr>
<th>percent</th>
<th>fraction</th>
<th>Decimal</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>37%</td>
<td>$\frac{37}{100}$</td>
<td>0.37 (37 hundredths)</td>
<td>37:100</td>
</tr>
</tbody>
</table>

Number sense with decimals and percents:

The following pattern might help us recognize decimal and percent equivalents.

0.0001 = 0.01%  
0.001 = 0.1%  
0.01 = 1%  
0.1 = 10%  
0.0025 = 0.25%  
0.25 = 25%  
2.5 = 250%

To convert a percent to a decimal - percent ÷ 100 = decimal

To convert decimal to percent - decimal x 100 = percent

Number sense with fractions and percents

When a fraction has a denominator 100, the numerator is the percent.

Ex. $\frac{45}{100} = 45\%$ and $\frac{350}{100} = 350\%$

Often it is easiest to write your fraction as a decimal and then as a percent.

Ex. $\frac{3}{8} = 3 \div 8 = 0.375 = 37.5\%$

Note: any fraction can be converted to a decimal using division

Decimal = numerator ÷ denominator

A ratio is another way to express the part out of 100. The ratio expresses numerator : denominator
Practice A – complete the following table

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/25</td>
<td></td>
<td>375%</td>
<td>3:5</td>
</tr>
<tr>
<td>8%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td></td>
<td>5:3</td>
</tr>
<tr>
<td>1/250</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Percent Problems

Consider the following

A full carton of eggs has 12 eggs.

So, 100% of 1 carton has 12 eggs.

Then 50% of 1 carton has 6 eggs because 50% x 12 eggs = 0.5 x 12 = 6 eggs

And so 150% of 1 carton is 18 eggs because 150% x 12 = 1.5 x 12 = 18 eggs

Note: percent x whole amount = part

Practice B

Calculate the following. When necessary, round your answer to the nearest tenth.

a. 28% of 40  
b. 3% of 20  
c. 234% of 8

d. 3½ % of 50  
e. 0.2% of 15 000  
f. 8.25% of 62
Other percent problems

Sometimes we might be given a part and asked to find the whole amount.

Ex 1. 22% of what number is 40? Ans: 40 ÷ 22% = 40 ÷ 0.22 =

Ex 2. 20% of all students walk to school. In my class, 4 students walk. How many students are in my class? Ans: 4 ÷ 20% = 4 ÷ 0.20 = 20

Or we may be given the part and whole and asked for the percent.

Ex 3. 25 out of 30 is what percent? Ans: 25 ÷ 30 = 0.83333... = 83.3%

Ex 4. 45 is what percent of 15? Ans: 45 ÷ 15 = 3 = 300%

The triangle will help you to determine if you need to multiply or divide to solve the problem.

\[
\begin{align*}
\text{a is the percent} & \quad \text{c = a\% x b} \\
\text{b is the whole} & \quad \text{b = c ÷ a\%} \\
\text{c is the part} & \quad \text{a\% = c ÷ b}
\end{align*}
\]

Practice C – Mixed problems – be sure to show your work!

1. What is 45% of 16?

2. 24 is what percent of 10?

3. 78% of what number is 22?

4. A salesman gets paid 35% commissions. How much commission does he make on sales of $700?

5. Mary has borrowed 48 books from the library. This is 22% of all of the books in the library. How many books are in the library?

6. Harry bought a shirt that was on sale for $29 after a 33% discount. What was the original price?

7. A cell phone case which regularly sells for $39 is on sale for 40% off. How much would you pay for this case, including NL sales tax?

*8. A box of cereal used to contain 450g. The new package has 400g? What is the percent decrease in the size of the package?

\[\text{Percent increase/decrease} = \frac{\text{change}}{\text{original}} \times 100\]
A ratio is a comparison of two or more quantities.

Ex. A hockey score of 3 to 2

- Ratios are usually written using a colon to separate the terms.

Ex. 3:2

- A ratio can have more than two terms.

Ex. A box contains 3 red, 4 green and 5 blue shirts.

The ratio of red to green shirts is 3:4

The ratio of green to red to blue shirts is 4:3:5

- There are many equivalent forms of any ratio, just as there are many equivalent forms of a fraction. Equivalent ratios are found by multiplying or dividing each term in a ratio by the same number.

Ex. \[2:3 = 4:6 = 6:9 = \ldots\]

Ex. \[45:100 = 9:20\]

- The lowest term ratios, highlighted above, are often the most useful.
- Note that when the second term of the ratio is 100, we have our percent.

Practice D

1. Complete the equivalent ratios.
   a. \[3:8 = 15:____\]
   b. \[9:18 = \_\_\_\_\_\_\_\_\_\_\_\_2\]
   c. \[1:2:3 = 6:____:____\]

2. Write each ratio in lowest terms.
   a. \[4:22\]
   b. \[160:24\]
   c. \[25:2500\]
   d. \[148:42:100\]

3. A sock drawer has 5 black socks, 12 white socks and 1 pink sock. Write the lowest term ratio of
   a. Black to white socks
   b. pink to all socks
   c. pink to black socks
   d. All socks to white socks

4. In # 3 above, what percent of socks are white?

5. Use proportions to solve the following problems.
   A. The ratio of ducks to ducklings ate Bowring park is 7:4. If there are 350 ducks, how many ducklings are there?
   B. The ratio of blue to red pens in my desk is 4:1. If there are 25 pens all together, how many are red and how many are blue?
   C. The scale on a nap is 1:50 000. If the distance on a map is 12cm, what is the actual distance, in km?
   D. The scale for a drawing is 10:1. If the actual length of a leaf is 3.4 cm, how long is its drawing?
A rate is used when you are comparing two quantities with different units.

Ex. 50 km in 2 hours.

A unit rate is when the second term of a rate is 1.

Ex. 25 km in 1 hour. Usually we say “25 km per hour” and we write 25km/hr

Ex. A 15 kg bag of potatoes costs $40. What is the unit cost?

\[
\text{Ans: } \frac{\$40}{15\text{ kg}} = \frac{\$40}{\text{15 kg} \div 15} = \frac{\$40}{\frac{15\text{ kg}}{1}} = \frac{\$40}{15} = 2.67\text{/kg}
\]

Ex. One athlete can ski 35km in 2hr. Another can ski 45 km in 2.5 hr. Which athlete is faster?

\[
\text{Ans: First athlete unit rate } = \frac{35\text{ km}}{2\text{ hr}} = \frac{35 \text{ km} \div 2}{2 \text{ hr} \div 2} = 17.5 \text{ km/hr}
\]

\[
\text{Second athlete unit rate } = \frac{45 \text{ km}}{2.5 \text{ hr}} = \frac{45 \text{ km} \div 2.5}{2.5 \text{ hr} \div 2.5} = 18 \text{ km/hr}
\]

The second athlete is faster!

Practice E

1. Tell if the following are examples of ratios or rates.
   a. Basketball score 45 to 50
   b. Scale on a map 1:10000
   c. Scale on a map 1cm:3km
   d. Apples $2.99/lb
2. Write each as a unit rate.
   a. 40 bars in 5 boxes  
   b. 48 slices in 4 pizzas  
   c. $8 for 5 kg
3. Which is the better buy?
   A. 3.5L juice for $7.49 or B. 1.5 L juice for $2.99
4. A diver descends 30m in 8 min. How long will it take her to descend 100m?
**Answers to Practice Questions**

**Practice A**

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{7}{1000}$</td>
<td>0.007</td>
<td>0.7%</td>
<td>7:1000</td>
</tr>
<tr>
<td>$\frac{12}{25}$</td>
<td>0.48</td>
<td>48%</td>
<td>12:25</td>
</tr>
<tr>
<td>$\frac{375}{100} = \frac{15}{4} = \frac{3}{4}$</td>
<td>3.75</td>
<td>375%</td>
<td>15:4</td>
</tr>
<tr>
<td>$\frac{3}{5}$</td>
<td>0.6</td>
<td>60%</td>
<td>3:5</td>
</tr>
<tr>
<td>$\frac{8}{100} = \frac{2}{25}$</td>
<td>0.08</td>
<td>8%</td>
<td>2:25</td>
</tr>
<tr>
<td>$\frac{250}{100} = \frac{5}{2} = 2\frac{1}{2}$</td>
<td>2.5</td>
<td>250%</td>
<td>5:2</td>
</tr>
<tr>
<td>$\frac{1}{250}$</td>
<td>0.004</td>
<td>0.4%</td>
<td>1:250</td>
</tr>
<tr>
<td>$\frac{5}{3}$</td>
<td>1.66666...</td>
<td>166.7%</td>
<td>5:3</td>
</tr>
</tbody>
</table>

**Practice B**

Calculate the following. When necessary, round your answer to the nearest tenth.

a. 28% of 40 = 0.28 x 40 = 11.2
b. 3% of 20 = 0.03 x 20 = 0.6

c. 234% of 8 = 2.34 x 8 = 18.72
d. 3½% of 50 = 0.035 x 50 = 1.75

e. 0.2% of 15 000 = 0.002 x 15 000 = 30
f. 8.25% of 62 = 0.0825 x 62 = 5.115

**Practice C**

1. $0.45 \times 16 = 7.2$
2. $24 \div 10 = 2.4 = 240%$
3. $22 \div 78\% = 22 \div 0.78 = 28.2$
4. $35\% \times 700 = 0.35 \times 700 = 245$
5. $48 \div 22\% = 48 \div 0.22 = 218.2$
6. 100% - 33% = 67% of the price was left after the sale.

$29 \div 67\% = 29 \div 0.67 = 43.3$
7. Sale Price L 40% off means 100% - 40% = 60% left. 60% of 39 = 0.60 x 39 = 23.40
Price with tax is 100% + 13% = 113% of sale price. 113% x 23.40 = 1.13 x 23.40 = $26.44

*8. 450 – 400 = 50 g change. 50 ÷ 450 = 0.11111 = 11.1%

Practice D

1. a. 3 : 8 = 15 : 40  b. 9 : 18 = 1 : 2  c. 1 : 2 : 3 = 6 : 12 : 18
2. a. 2:11  b. 20:3  c. 1:100  d. 74:21:50
3. a. 5:12  b. 1:18  c. 1 : 5  d. 18:12 = 3:2
4. 12 ÷18 = 0.66666 = 66.7%
5. A. 7 : 4 = 350 : x, 7 x 50 = 350 so x = 4 x 50 = 200 ducklings
   B. 4 blue + 1 red = 5 pens all together. 4 blue:5 pens = b:25 pens, 5 x 5 = 25 so b = 4 x 5 = 20 blue pens. 25 pens – 20 blue = 5 red pens
   C. 1 : 50 000 = 12 cm : d, 50 000 x 12 cm = 600 000 cm = 60 km
   D. 10: 1 = d: 3.4 cm, 10 x 3.4 = 34 cm long drawing

Practice E

1. a. Ratio  b. Ratio  c. Rate  d. Rate
2. a. \( \frac{40 \text{ bars}}{5 \text{ boxes}} \div 5 = 8 \text{ bars/box} \)  b. \( \frac{48 \text{ slices}}{4 \text{ pizzas}} \div 4 = 12 \text{ slices/pizza} \)  c. \( \frac{58 \div 5}{5 \text{ kg}} \div 5 = \$1.60/\text{kg} \)
3. A. $7.49 ÷ 3.5 L = $2.14/L  B. $2.99 ÷ 1.5L = $1.99/L.  B is cheaper and the better buy
4. \( \frac{30 \text{ m} + 30}{8 \text{ min} + 30} = \frac{1 \text{ m}}{0.27 \text{ min}} \quad \frac{1 \text{ m} \times 100}{0.27 \text{ min} \times 100} = \frac{100 \text{ m}}{27 \text{ min}} \)